



FSH-PH Publications

BASIC CALCULUS

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CHAPTER 1

LIMITS AND CONTINUITY OF FUNCTIONS

Learning Competencies

At the end of the chapter, the students learn to:

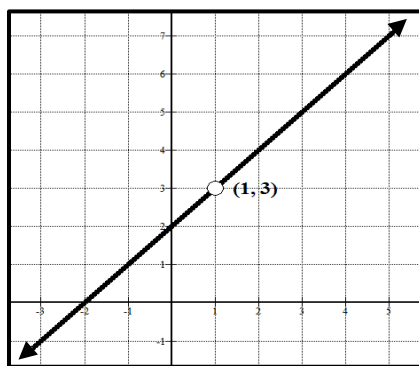
- ✚ Illustrate the limit of a function using a table of values and the graph of the function.
- ✚ Distinguish between $\lim_{x \rightarrow a} f(x)$ and $f(a)$.
- ✚ Illustrate the limit laws.
- ✚ Apply the limit laws in evaluating the limit of algebraic functions (polynomial, rational, radical).
- ✚ Compute the limits of exponential, logarithmic and trigonometric functions using table of values and graphs of the functions.
- ✚ Evaluate limits involving $\frac{\sin x}{x}$, $\frac{1-\cos x}{x}$ and $\frac{e^x-1}{x}$ using table of values.
- ✚ Illustrate continuity of a function at a number.
- ✚ Determine whether a function is continuous at a number or not.
- ✚ Illustrate continuity of a function on an interval.
- ✚ Determine whether a function is continuous on an interval or not.
- ✚ Illustrate different types of discontinuity (removable, jump, infinite).
- ✚ Illustrate the Intermediate Value and Extreme Value Theorems.
- ✚ Solve problems involving continuity of a function.

1.1. Limit of a Function

In Calculus, the idea of limit is very important. The concept of limit is at the foundation of almost all mathematical analysis and an understanding of it is absolutely essential. Deep understanding of limit is very rewarding since it facilitates a good grasp of all the basic processes of Calculus.

Let us consider a particular function, say $f(x) = \frac{x^2 + x - 2}{x - 1}$. This function is defined for all values of x except at $x = 1$. This is because both the numerator and denominator of function $f(x)$ take zero value at $x = 1$, that is, $f(1) = \frac{0}{0}$, a meaningless expression or is an indeterminate form, hence, making one not an element of the domain of $f(x)$. However, factoring $f(x) = \frac{x^2 + x - 2}{x - 1}$ reduces it to

$f(x) = \frac{(x+2)(x-1)}{x-1} = x+2$, provided $x \neq 1$. Note on the straight line graph of $f(x) = y = x+2$ shown below, point $(1,3)$ does not lie on the graph, hence, an open circle or a hole is seen on its graph.



Let us study how the function f behaves when we assume values of x getting closer and closer to 1. This means x nears 1 but never equal to 1. There are two ways by which value of x may approach 1. One is by assuming values of x less than 1 (or at the left of 1) but approaching 1. The other way is by assuming values of x more than 1 (or at the right of 1) but also approaching 1.

To get a better view of what happens as x takes values approaching 1 but less than 1 ($x < 1$), let us consider the table of values on the succeeding discussions.

From Table 1 on the next page, observe that the value of function $f(x) = \frac{x^2 + x - 2}{x - 1}$ approaches 3 as x takes values less than 1 but approaching 1. Note that as x gets closer and closer to 1 through values less than 1, the value of $f(x)$ gets closer and closer to 3; and the closer x is to 1, the closer $f(x)$ is to 3. We can make the value of

$f(x)$ or the y value as close to 3 as we please by taking x close enough to 1. Another way of saying this is that we can make the absolute value of the difference between $f(x)$ and 3 as small as we please by making the absolute value of the difference between x and 1 small enough. That is, $|f(x) - 3|$ can be made as small as we please by making $|x - 1|$ small enough. But bear in mind that $f(x)$ never takes on the value 3. That is, provided $x \neq 1$, as $x \rightarrow 1^-$, $f(x) \rightarrow 3$. This is read “as x approaches 1 through values less than 1, $f(x)$ approaches 3”. In symbol form, $\lim_{x \rightarrow 1^-} f(x) = 3$ read “limit of $f(x)$ as x approaches 1 through values less than 1 is equal to 3”. In particular, this limit is called the *Left-hand Limit* of $f(x)$.

Table 1

x	$f(x) = \frac{x^2 + x - 2}{x - 1}$
0.90	2.9
0.99	2.99
0.999	2.999
0.9999	2.9999
0.99999	2.99999
0.999999	2.999999
0.9999999	2.9999999

1.2. Definition of Left-hand Limit of a Function

The notation for the Left-Hand Limit of $f(x)$ is $\lim_{x \rightarrow a^-} f(x) = L$.

Here, $f(x)$ is made close to L for all x sufficiently close to a and $x < a$ without actually letting x be a .

Let us now look at Table 2 where the values taken by x approach 1 through values greater than 1 ($x > 1$).

Table 2

x	$f(x) = \frac{x^2 + x - 2}{x - 1}$
1.1	3.1
1.01	3.01
1.001	3.001
1.0001	3.0001
1.00001	3.00001
1.000001	3.000001
1.0000001	3.0000001

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